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Quantum Logic Gates and Cluster States with Four-level Atoms in Cavity QED

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Abstract We propose a model to implement the two-qubit quantum logic gates, i.e., the quantum phase gate and the Controlled-NOT gate, and generate the atomic qubits cluster states with a large detuned interaction between four-level atoms and a single-mode cavity field. In the presented protocol, the quantum information is encoded on the stable ground states of the atoms, and the effect of decoherence from atomic spontaneous emission is negligible. In addition, the interaction between atoms and the cavity is large detuned, and the cavity is only virtually excited. Therefore, the scheme is insensitive to the cavity decay. The experimental feasibility of our proposal is also discussed.

Keywords Cavity QED · Quantum logic gates · Cluster states

1 Introduction

Cavity quantum electrodynamics (QED) is an ideal candidate for implementing quantum information processing [1]. The reason is based on the following two points. (i) Photons are ideal carriers for fast and reliable communication over long distances, and the atoms are good memorizers for storing and processing quantum information. Thus the combination of atoms and photons can be useful in quantum computation. (ii) The atoms trapped in a high-Q cavity have long decoherence time [2]. On the other hand, the two-qubit quantum logic gates are the building blocks of computers. Therefore, it has been attracting much attention to find efficient ways to implement two-qubit quantum gates in cavity QED. Lately, several schemes for implementing two-qubit logic gates have been proposed [3–5]. But the influence of the atomic spontaneous emission may decrease the experimental feasibility of their schemes.

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Quantum computation can also be achieved by local measurements and feedforward of their outcomes with cluster states [6]. The cluster states are a typical class of genuine multipartite entangled states, and have a larger persistency of entanglement than the well-known Greenberger-Horne-Zeilinger (GHZ) states [7]. Recently, the study on cluster states and their applications has attracted much interest. It has been shown that a new inequality is maximally violated by the four-particle cluster states, but not the four-particle GHZ states [8]. Schemes for implementing quantum communication via cluster states were also proposed [9]. Because of their novel properties and extensive applications, the preparation of the cluster states has become a new research topic. Up to now, many schemes for generating cluster states have been proposed in the context of cavity QED [10–21]. But in their schemes, the unavoidable decoherence influences due to spontaneous emission of the excited states or decay of the cavity modes will largely decrease the fidelity for experimental implementation.

In this paper, we construct an atom-cavity interaction model which is insensitive to both the cavity decay and atomic spontaneous emission. This model can be used to implement basic two-qubit quantum logic gates and generate atomic qubit cluster states. Our proposal has a distinct advantage: the qubits are encoded on the ground states of the atoms which have relatively long lifetime.

2 Interaction of the Atoms with the Cavity-Field

The system considered here consists of two four-level atoms simultaneously passing through a single-mode cavity-field. The level structure of the atomic configuration is shown in Fig. 1. Each atom has three stable ground states $|l>$, $|g>$, and $|s>$, and a excited state $|e>$. The coherence time of the atomic ground levels are so long that these states can be used to store quantum information. The atomic transition $|g>\to|e>$ ($|s>\to|e>$) is dispersively coupled to a single quantized cavity mode (a classical laser field) with coupling constants $g(\Omega_L)$ and detuning $\Delta_1(\Delta_2)$, while the atomic ground level $|l>$ is not affected by the cavity and classical field.

Taken the laser field into account, the system Hamiltonian can be given by

$$H=\sum_{j=1}^{2}(g_j a\,|e_j><g_j|\,e^{-i\Delta_1 t}+\Omega_{Lj}\,|e_j><s_j|\,e^{-i\Delta_2 t}+H.c.), \tag{1}$$

where g_j represents coupling strength between the jth atom and the cavity, a^+ and a are the creation and annihilation operators of the quantized cavity mode, respectively. Without loss of generality, we assume Δ_i, Ω_{Lj} and g_j are real. When $\Delta_2 \gg \Omega_{Lj}, \Delta_1 \gg g_j$, we can eliminate adiabatically the excited energy level $|e>$ by neglecting the effect of rapidly oscillating terms. Then the above Hamiltonian reduces to [22–24]

$$H_E=\sum_{j=1}^{2}\left[\frac{g_j^2}{\Delta_1}a^+a\,|g_j><g_j|+\frac{\Omega_{Lj}^2}{\Delta_2}\,|s_j><s_j|+\frac{g_j\Omega_{Lj}}{\Delta}(ae^{i\delta t}\,|s_j><g_j|+H.c.)\right], \tag{2}$$

where $\Delta=\frac{2\Delta_1\Delta_2}{\Delta_1+\Delta_2}$, $\delta=\Delta_2-\Delta_1$. The first two terms in (2) describe the Stark shifts of the levels $|g>$ and $|s>$ that are induced by the cavity mode and the external classical laser field. The last two terms describe the effective atomic interaction. For simplicity, set $g_j=g$ and $\Omega_{Lj}=\Omega_L$ in the following calculation. Under the large-detuning limit conditions

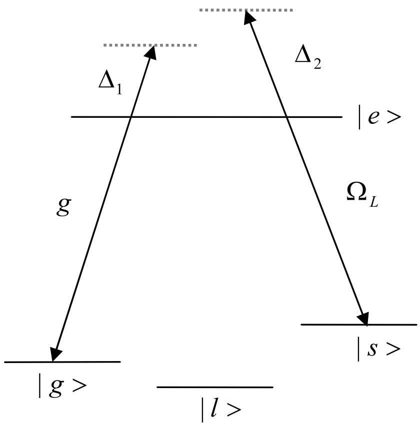

Fig. 1 Involved level structure of the four-level atom and the corresponding atomic transition

$\delta \gg \frac{g\Omega_L}{\Delta}$, the atomic system cannot exchange energy with the cavity-mode. Then the only energy-conversing transition is between $|s_j, g_k, n>$ and $|g_j, s_k, n>$, which is mediated by the energy levels $|g_j, g_k, n+1>$ and $|s_j, s_k, n-1>$, independent of photon-number states of the cavity mode, and the Rabi frequency is $\eta = \frac{g^2\Omega_L^2}{\Delta^2\delta}$. Then the effective Hamiltonian of the (2) can be written as [25, 26]

$$H_E^{'} = \sum_{j=1}^{2}\left(\frac{g^2}{\Delta_1}a^+a\,|g_j><g_j| + \frac{\Omega_L^2}{\Delta_2}|s_j><s_j|\right)$$
$$+\eta\sum_{j=1}^{2}(|s_j><s_j|aa^+ - |g_j><g_j|a^+a)$$
$$+\eta\sum_{j,k=1,j\neq k}^{2}(|s_j><g_j|\otimes|g_k><s_k| + |g_j><s_j|\otimes|s_k><g_k|). \quad (3)$$

The third and fourth terms describe the photon-number dependent Stark shifts induced by the off-resonant driving, and the last two terms describe the dipole coupling between the atoms mediated by the cavity mode and the classical microwave pulse. Assuming the cavity field is initially in the vacuum state, the effective Hamiltonian then reduces to

$$H_{eff} = \sum_{j=1}^{2}\left(\frac{\Omega_L^2}{\Delta_2} + \eta\right)|s_j><s_j|$$
$$+\eta\sum_{j,k=1,j\neq k}^{2}(|s_j><g_j|\otimes|g_k><s_k| + |g_j><s_j|\otimes|s_k><g_k|). \quad (4)$$

According to the effective Hamiltonian, we can obtain the following state-evolution via solving Schrödinger equation $i\partial_t|\Psi(t)>= H_{eff}|\Psi(t)>$,

$$\begin{aligned}
&|g_1>|g_2>\rightarrow|g_1>|g_2>,\\
&|g_1>|l_2>\rightarrow|g_1>|l_2>,\\
&|s_1>|l_2>\rightarrow e^{-i(\frac{\Omega_L^2}{\Delta_2}+\eta)t}|s_1>|l_2>,\\
&|s_1>|g_2>\rightarrow e^{-i(\frac{\Omega_L^2}{\Delta_2}+\eta)t}(\cos\eta t\,|s_1>|g_2> - i\sin\eta t\,|g_1>|s_2>).
\end{aligned} \quad (5)$$

3 Realization of Two-qubit Quantum Logic Gates

We now begin to analyze how to realize two-qubit quantum logic gates using state-evolution in (5). By choosing the parameters $\frac{\Omega_L^2}{\Delta_2} = 2k\eta$ and $t = \pi/\eta$ ($k \in$ integer), it is easy to check that a controlled-phase-gate-like phase factor π will be obtained:

$$\begin{aligned}
&| g_1 >| g_2 > \to | g_1 >| g_2 >,\\
&| g_1 >| l_2 > \to | g_1 >| l_2 >,\\
&| s_1 >| g_2 > \to | s_1 >| g_2 >,\\
&| s_1 >| l_2 > \to -| s_1 >| l_2 > .
\end{aligned} \tag{6}$$

Next we will use this phase gate to implement a quantum Controlled-NOT gate. We assume atom 2 acts as the target atom, while atom 1 acts as the control atom. In order to realize a Controlled-NOT gate, the scheme consists of three-step operations.

Step (I): Let atom 2 pass sequentially through two classical fields tuned to the transitions $| g > \to | s >$ and $| s > \to | l >$. If we appropriately adjust the amplitude and phase of the classical fields, make atom 2 undergo the transformation

$$\begin{aligned}
&| s_2 > \to (| s_2 > + | g_2 >)/\sqrt{2} \to (| l_2 > + | g_2 >)/\sqrt{2},\\
&| g_2 > \to (| g_2 > - | s_2 >)/\sqrt{2} \to (| g_2 > - | l_2 >)/\sqrt{2}.
\end{aligned} \tag{7}$$

Step (II): Make the two atoms simultaneously cross the high-Q single-mode optical cavity, the time evolution of the basis states is decided by the effective Hamiltonian. Then the evolutions in the form of (6) will take place.

Step (III): Let atom 2 enter two classical field tuned to the transitions $| l > \to | s >$, and $| g > \to | s >$. If we appropriately adjust the amplitude and phase of the classical fields, make atom 2 undergo the transformations

$$\begin{aligned}
&| g_2 > \to (| s_2 > + | g_2 >)/\sqrt{2},\\
&| l_2 > \to | s_2 > \to (| s_2 > - | g_2 >)/\sqrt{2}.
\end{aligned} \tag{8}$$

After the three steps, a Controlled-NOT gate can be obtained as follows

$$\begin{aligned}
&| g_1 >| g_2 > \to | g_1 >| g_2 >,\\
&| g_1 >| s_2 > \to | g_1 >| s_2 >,\\
&| s_1 >| g_2 > \to | s_1 >| s_2 >,\\
&| s_1 >| s_2 > \to | s_1 >| g_2 > .
\end{aligned} \tag{9}$$

In our protocol, it is easy to see that the first and third steps are not affected by the interaction between the two atoms and the single-mode vacuum cavity-field.

4 Preparation of Atomic Cluster States

In this section, we will show how the N-qubit cluster states can be prepared in the N-atom case. In order to generate an N-atom cluster state, we prepare atom 1 in the state $\frac{1}{\sqrt{2}}(|g_1>+|s_1>)$, others in $\frac{1}{\sqrt{2}}(|g_j>+|l_j>)(j=2,3,\ldots,N)$, and cavity modes in the vacuum states $|0>$. So the initial state of the system is

$$|\Psi>=\frac{1}{2^{N/2}}(|g_1>+|s_1>)\bigotimes_{j=2}^{N}(|g_j>+|l_j>). \tag{10}$$

Firstly, let atoms 1 and 2 simultaneously cross a high-Q cavity. They will undergo the evolution of (6) without interaction with any other atom. This leads (10) to

$$|\Psi>=\frac{1}{2^{N/2}}[(|s_1>\otimes(|g_2>-|l_2>)+|g_1>\otimes(|g_2>+|l_2>)]\bigotimes_{j=3}^{N}(|g_j>+|l_j>). \tag{11}$$

After that, apply a single-qubit rotation on atom 2 through an auxiliary classical field $|l_2>\leftrightarrow|s_2>$. Then, the total system is in the state

$$|\Psi>=\frac{1}{2^{N/2}}(|s_1>\sigma_z^2+|g_1>)(|g_2>+|s_2>)\bigotimes_{j=3}^{N}(|g_j>+|l_j>), \tag{12}$$

where $\sigma_z^2=|g_2><g_2|-|s_2><s_2|$. In the same way, for atom $j(j<N)$, we repeat the above operations on the atoms j and $j+1$ and end the process when $j=N$. Then we obtain N-qubit cluster states [10, 27]

$$|\Psi>=\frac{1}{2^{N/2}}\bigotimes_{j=1}^{N}(|s_j>\sigma_z^{j+1}+|g_j>), \tag{13}$$

where $\sigma_z^{N+1}\equiv 1$. It should be noted that the operations on the neighbor atom pairs, e.g., operations on atom pairs $(j,j+1)$ and $(j+2,j+3)$, can be simultaneously performed [18]. Thus the N-atom cluster states can be prepared at a high speed, as two steps can achieve the goal.

Next, we introduce another scheme for preparing multi-atom cluster states by using the same four-level atoms system. Compared with the first scheme, the two-atom interaction time of this scheme is only half of the first scheme. We prepare atom 1 in the state $\frac{1}{\sqrt{2}}(|l_1>-|s_1>)$, the second atom in the state $|g_2>$, and the cavity-fields occupancy the vacuum states $|0>$. So the initial state of the system is $|\Psi>_0=\frac{1}{\sqrt{2}}(|l_1>-|s_1>)|g_2>$.

Firstly, we simultaneously send atom 1 and 2 across the single-mode cavity, the state evolution is described by (5). After the interaction time $t=\pi/2\eta(\text{set}\frac{\Omega_L^2}{\Delta_2}=4\eta)$, $|\Psi>_0$ becomes

$$|\Psi>=\frac{1}{\sqrt{2}}(|l_1>|g_2>+|g_1>|s_2>). \tag{14}$$

Then, we can use one classical field tuned to the transitions $|s>_2\leftrightarrow|g>_2$. The amplitude and the phase of the classical field are chosen appropriately, so that the second atom undergoes the transitions

$$\begin{aligned}|s_2>&\rightarrow(|g>_2-|s>_2)/\sqrt{2},\\|g>_2&\rightarrow(|g>_2+|s>_2)/\sqrt{2}.\end{aligned} \tag{15}$$

Then the state becomes $|\Psi>=\frac{1}{2}(|g_2>+|s_2>\sigma_Z^1)(|l_1>+|g_1>)$ with $\sigma_z^j=|l_j><l_j|-|g_j><g_j|$ and the subscript indicates the Pauli operator of the jth atom. After that, two classical fields are used to induce the transitions $|s_2>\to|l_2>\to|l_2>$ and $|g_2>\to|g_2>\to-|s_2>$. Obviously two-atom cluster state is obtained as follows

$$|\Psi>=\frac{1}{2}(|l_2>\sigma_Z^1-|s_2>)(|l_1>+|g_1>). \tag{16}$$

If the initial state of the system is $|\Psi>=\frac{1}{\sqrt{2}}(|l_1>-|s_1>)|g_2>|g_3>$. By repeating the above-mentioned procedure from (14) to (16) on the jth ($j\in 2,3$) atom (choosing the same time $t=\pi/2\eta$), and the different classical fields tuned to the transitions $|s_j>\leftrightarrow|g_j>$, $|s_j>\to|l_j>\to|l_j>$ and $|g_j>\to|g_j>\to-|s_j>$ in turn. After performing the similar operations, the state of the atoms becomes

$$\begin{aligned}|\Psi> &= \frac{1}{2\sqrt{2}}(|l_3>\sigma_Z^2-|s_3>)(|l_2>\sigma_Z^1+|g_2>)(|l_1>+|g_1>)\\ &= \frac{1}{2^{3/2}}(|l_3>\sigma_Z^2-|s_3>)\bigotimes_{j=1}^{2}(|l_j>\sigma_Z^{j-1}+|g_j>),\end{aligned} \tag{17}$$

where $\sigma_z^0\equiv 1$. The three-atom cluster state is in the form of (17).

By this method, a multi-atom cluster state can be generated if the initial state of the N-atoms system is

$$|\Psi>=\frac{1}{\sqrt{2}}(|l_1>-|s_1>)|g_2>\cdots|g_N>. \tag{18}$$

Every pair of neighbor-labeled atoms is implemented on the manipulations mentioned above, then we can obtain [18]

$$|\Psi>=\frac{1}{2^{N/2}}(|l_j>\sigma_z^{N-1}-|s_j>)\bigotimes_{j=1}^{N-1}(|g_j>+|l_j>\sigma_z^{j-1}), \tag{19}$$

where $\sigma_z^0\equiv 1$. The state (19) is just an N-atom cluster state. In order to transform the state (19) into the general form

$$|\Psi>=\frac{1}{2^{N/2}}\bigotimes_{j=1}^{N}(|g_j>+|l_j>\sigma_z^{j-1}). \tag{20}$$

We can use the classical pulse to make the Nth atom undergo the transitions $|s_N>\to-|g_N>$.

Note that the manipulations on labeled neighbor odd number (or even number) pair atoms can be simultaneously implemented [18]. In this sense, the presented scheme may take less time than the previous ones [10, 14] in the experiments for the same purpose of preparing cluster states. Furthermore, the second scheme works with the assistance of additional single-qubit rotations, but it can save the two-atom interaction time, as its two-atom operation time ($t=\pi/2\eta$) is half of the former one's ($t=\pi/\eta$).

5 Discussion and Conclusion

We now give a brief discussion of the experimental feasibility of the proposed scheme within current cavity QED technology. Firstly, we can take ^{87}Rb to meet the required level structures for our proposal [22–24]. The ground states $|l>$, $|g>$ and $|s>$ may correspond to

Zeeman levels of $|F=1,m=0>$ of $5S_{1/2}$, $|F=2,m=2>$ of $5S_{1/2}$ and $|F=2,m=3>$ of $5S_{1/2}$, while the excited state $|e>$ is denoted by the Zeeman level $|F=2,m=2>$ of the $5P_{1/2}$. We choose the experimental parameter $\Omega_L=0.408g$, $\Delta_1=10g$, $\Delta_2=10.4g$ so that $\delta=0.4g$ and $\frac{g^2}{\Delta_1}=0.1g$, $\frac{\Omega_L^2}{\Delta_2}=0.016g$, $\eta=\frac{g^2\Omega_L^2}{\Delta^2\delta}=0.004g$. It is easy to check that the large detuning conditions $\Delta_i\gg\{\frac{g^2}{\Delta_1},\frac{\Omega_L^2}{\Delta_2}\}$, $\delta\gg\frac{g\Omega_L}{\Delta}$ and $\frac{\Omega_L^2}{\Delta_2}=4\eta$ can be well satisfied. For a practical task of computation on the one-way quantum computer, however, such one-dimensional cluster states are not sufficient, as their geometry does not permit the realization of two-qubit gates. Therefore, it is worth pointing out that our model can be applied to generate two-dimensional cluster states easily using the same procedure of 2D atomic qubits cluster states in the Ref. [18]. In addition, naturally, the phase gate of our scheme can also be utilized to prepare the two-dimensional cluster states.

In summary, we have considered cavity QED schemes to realize basic two-qubit quantum logic gates, i.e., the quantum phase gate and the Controlled-NOT gate, in cavity QED system through a large detuned atom-cavity interaction with four-level atoms. In particular, two schemes for generating the atomic cluster states have been proposed with the presented gate operation. The stable ground states of the atoms are encoded as qubits, and the schemes are insensitive to both the spontaneous emission of the excited states and decay of the cavity modes. The interaction time between atoms and cavities is very short and only the ground state of atom is utilized, which has lower energy than the other two atomic states and can be relatively stable in the experiment.

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